Toward an Approximation Theory for Computerised Control

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- Problem statement
- Main result
- Conclusion and questions

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sampling theory + numerical analysis + stability

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What about mixed (hybrid) systems ?

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  $\Rightarrow$ Needs for a sampling theory of discrete event and hybrid systems

  or more generally an approximation theory of discrete event and hybrid systems

What do we mean by a theory?

Verification or synthesis algorithms?

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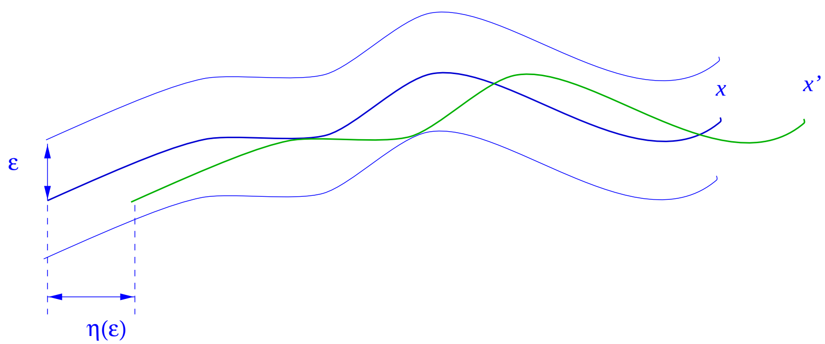

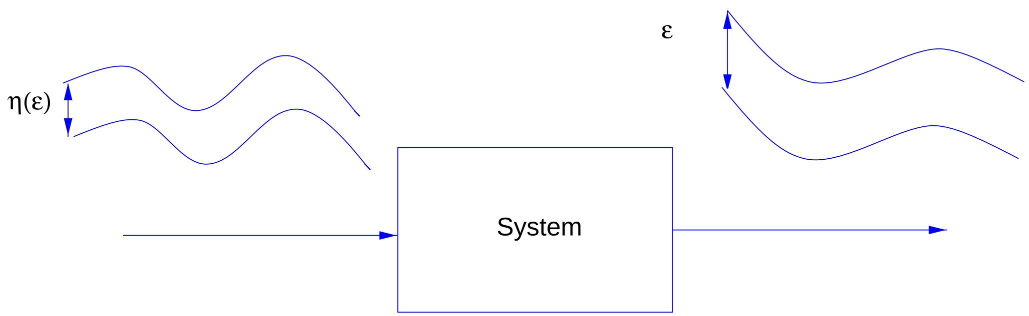

Proof based on the L_∞ norm

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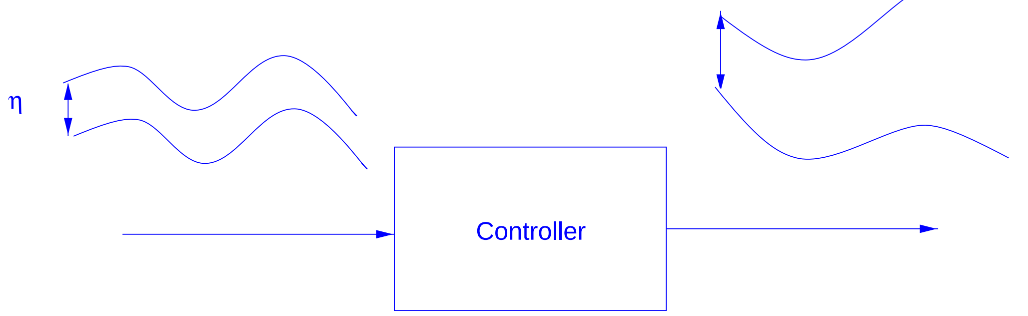
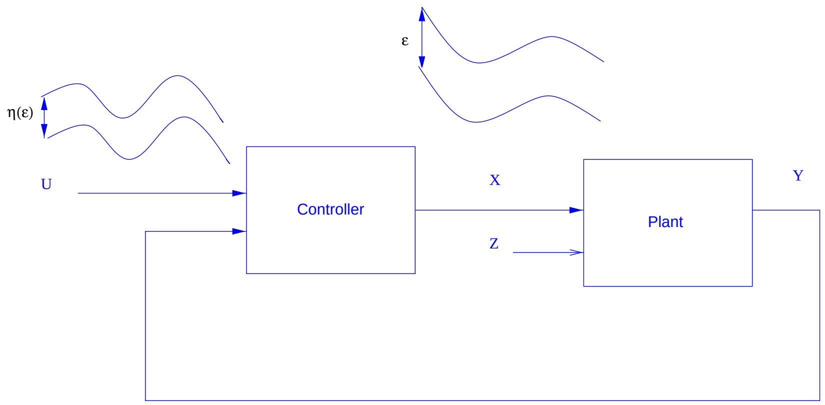
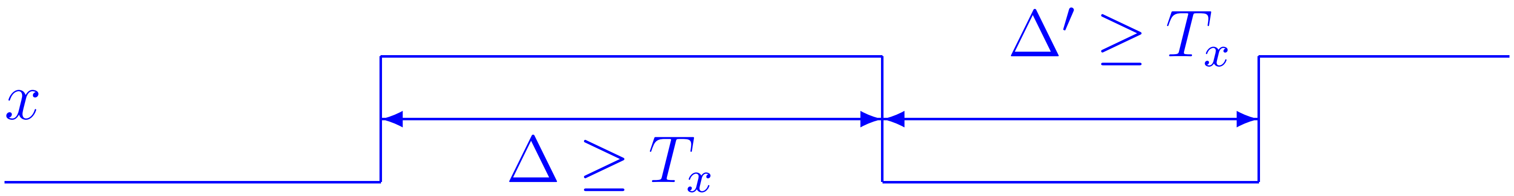




Main Result

Samplable boolean signals are signals with uniform bounded variability:

There exists a minimum stable time T_x associated with a signal x.



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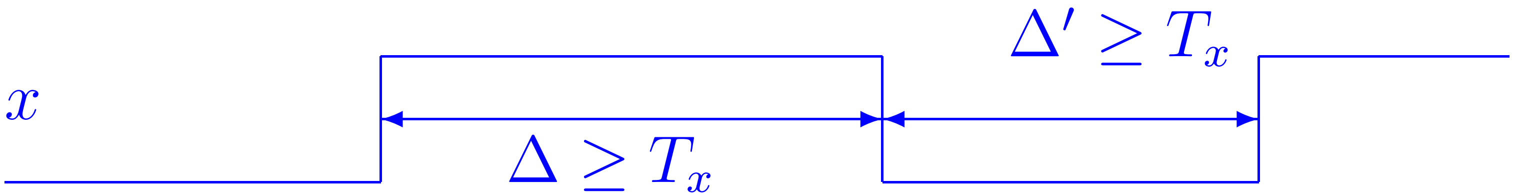

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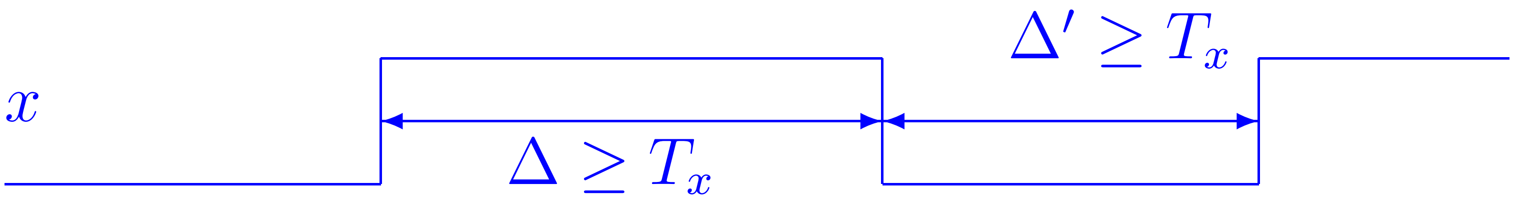

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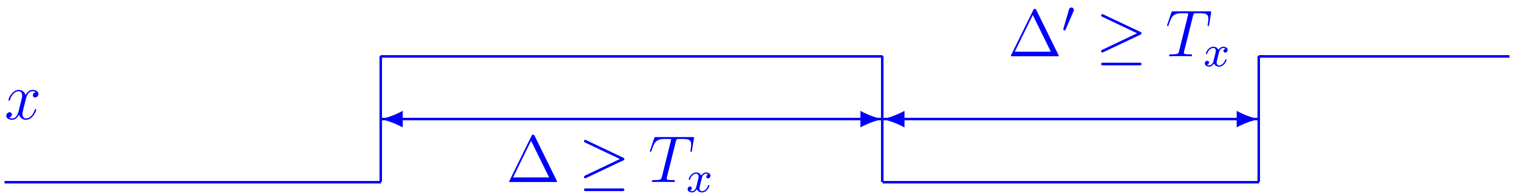

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Topology is recovered and this encompasses both continuous, discrete and piece-wise continuous signals.

Skorokhod Distance

Based on bijective retiming:

- retiming r : non decreasing mapping from $\mathcal{R}^+$ to $\mathcal{R}^+$
- bijective retiming $\Rightarrow$ increasing and continuous

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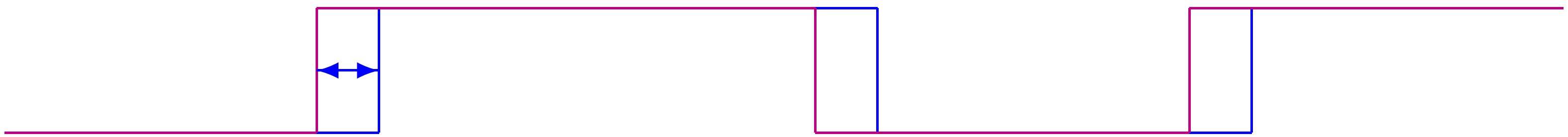
Boolean signal example : the largest time shift between corresponding edges



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More generally, the best compromise between shifts and errors

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On the contrary, for unboundedly fast varying signals, a non continuous retiming can erase some discontinuities while a continuous one cannot : compensation is no more possible

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